

# Generalized Quantum Telecloning

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We show a generalized telecloning (GTC) protocol where the quantum channel is non-optimally entangled and each channel qubit undergoes a different decoherence process. We show that one can increase the fidelity of telecloned states by properly choosing the measurement basis at Alice's, albeit turning the protocol to a probabilistic one. We also show how one can convert the GTC protocol to the teleportation protocol via proper unitary operations.

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## I. INTRODUCTION

Since the introduction of the quantum teleportation protocol [1] and its experimental demonstration [2, 3], whereby an arbitrary state describing a quantum system can be transferred from one recipient (Alice) to another (Bob), several new quantum communication protocols have appeared. They allow the sharing of quantum states among several recipients [4], the sharing of quantum secrets [5, 6, 7], or the teleportation of an arbitrary quantum state to many recipients, i.e quantum telecloning [8]. The latter protocol does not violate the no-cloning theorem [9] since the fidelity of the telecloned states with respect to the original one are not perfect, and decreases with the number of copies. An optimal quantum telecloning protocol has been presented in Refs. [8, 10] for two-level systems (qubits) and later on quantum telecloning has been demonstrated experimentally for continuous variables systems [11, 12].

These protocols are essential to many quantum information tasks which require a secure transmission of quantum states. One example is quantum information networks [4, 13], which are built of nodes in which quantum states are created, manipulated, and stored. These nodes are connected by multipartite entangled quantum channels and by properly using one or several of the aforementioned protocols one could avoid errors and eavesdropping during the transmission of a state between nodes [4, 14].

However, most treatments of these protocols assume bipartite or multipartite maximally entangled channels, whereas in realistic scenarios decoherence and noise ensure that that is not the case. One suggested solution is quantum distillation protocols [15], which allow us to obtain a maximally entangled state from a large ensemble of partially entangled states, although only asymptotically. Another one is to dynamically control the decoherence of

the channel qubits [16, 17].

In Ref. [18], inspired by Ref. [19], and in Ref. [20], we have generalized the teleportation [18] and quantum state sharing [20] protocols to an arbitrary number of input qubits and shown that one can overcome the fidelity decrease due to non-maximally entangled channels on expense of transforming the protocols to probabilistic ones. These generalized protocols give the parties freedom to allocate the channel's resources to a continuous distribution between the fidelity of the protocol and its probability of success to achieve a given fidelity.

In this contribution we present the generalized telecloning protocol (GTC), where we generalize the standard quantum telecloning protocol to non-optimally entangled multipartite channels (see Fig. 1). By treating each qubit's degraded contribution to the entangled channel separately, we show that one can overcome the resulting fidelity decrease by applying appropriate modifications to the protocol. Our main results show that: (a) the port's qubit decoherence effects can be overcome by changing the measurement basis; (b) the ancillary qubit's decoherence have no effect on the telecloned fidelity; (c) the copy qubits' decoherence have a non trivial influence on the fidelity and we show the optimal strategy to maximize the efficiency of the protocol; and (d) it is possible to convert the GTC to the generalized teleportation protocol (GTP) if one allows Alice to implement certain types of unitary operations on the channel's qubits.

## II. GENERAL FORMALISM

We focus our attention on the “ $1 \rightarrow 2$  quantum telecloning”, i.e. one original qubit and two copies. Let us assume that Alice wishes to teleclone her state to Bob and Charlie. The quantum channel used for the optimal telecloning protocol [8, 10] is composed of four qubits, namely port qubit, ancillary qubit and two copy qubits. The port and ancillary qubits are assumed to be with Alice, although the ancillary are not required to be there [8]. One copy qubit is with Bob while the other one is

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with Charlie (Fig. 1).

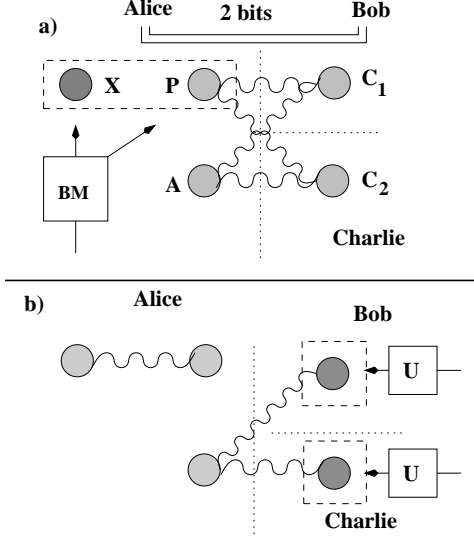


FIG. 1: Alice performs a Bell measurement (BM) on the qubit to be telecloned (X) and on the port qubit (P). She then tells Bob and Charlie her measurement result (2 bits). The copies  $C_1$  and  $C_2$  are then subjected to a proper unitary operation (U). Note that waves represent the existence of pairwise entanglement among the qubits and that the ancillary qubit (A) is entangled with the copies at the end of the protocol.

The channel state is given by:

$$|\psi\rangle_{PAC} = \frac{1}{\sqrt{2}} (|0\rangle_P \otimes |\phi_0\rangle_{AC} + |1\rangle_P \otimes |\phi_1\rangle_{AC}), \quad (1)$$

where

$$|\phi_0\rangle_{AC} = \sum_{j=0}^1 \alpha_j |\{1-j\}\rangle_A \otimes |\{2-j\}\rangle_C, \quad (2)$$

$$|\phi_1\rangle_{AC} = \sum_{j=0}^1 \alpha_j |\{j\}\rangle_A \otimes |\{j\}\rangle_C, \quad (3)$$

$$\alpha_j = \sqrt{(2-j)/3}. \quad (4)$$

Here the subscripts denote the port (P), ancillary (A) and copies (C:  $C_1$  with Bob and  $C_2$  with Charlie). The state  $|\{M-j\}\rangle$  denotes the symmetric and normalized state of  $M$  qubits where  $M-j$  of them are in state  $|0\rangle$  and  $j$  are in the orthogonal state  $|1\rangle$ . For  $M=2$  we have explicitly,

$$\begin{aligned} |\phi_0\rangle_{AC} &= \sqrt{\frac{2}{3}} |000\rangle_{AC} + \sqrt{\frac{1}{6}} |101\rangle_{AC} + \sqrt{\frac{1}{6}} |110\rangle_{AC}, \\ |\phi_1\rangle_{AC} &= \sqrt{\frac{2}{3}} |111\rangle_{AC} + \sqrt{\frac{1}{6}} |001\rangle_{AC} + \sqrt{\frac{1}{6}} |010\rangle_{AC}. \end{aligned}$$

We introduce decoherence by applying a qubit-specific decoherence operator:

$$\hat{D}_i (\alpha|0\rangle_i |\psi_0\rangle + \beta|1\rangle_i |\psi_1\rangle) = \frac{\alpha|0\rangle_i |\psi_0\rangle + n_i \beta|1\rangle_i |\psi_1\rangle}{\sqrt{|\alpha|^2 + |n_i \beta|^2}}, \quad (5)$$

where  $n_i$  can be complex and  $|\alpha|^2 + |\beta|^2 = 1$ . For example, when this operator is applied on a Bell state, e.g.  $|\Phi^+\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$ , it produces a non-maximally entangled state,  $\hat{D}_1(|\Phi^+\rangle) = 1/\sqrt{1+|n_1^2|}(|00\rangle + n_1|11\rangle)$ .

Applying this operator on each qubit in the telecloning channel results in:

$$|\psi; \{n\}\rangle_{PAC} = A \left( |0000\rangle + \frac{n_P n_{C_1}}{2} |1010\rangle + \frac{n_A n_{C_1}}{2} |0110\rangle + \frac{n_P n_{C_2}}{2} |1001\rangle + \frac{n_A n_{C_2}}{2} |0101\rangle + n_P n_A n_{C_1} n_{C_2} |1111\rangle \right)_{PAC}, \quad (6)$$

where

$$A = \left( 1 + \frac{|n_P n_{C_1}|^2}{4} + \frac{|n_A n_{C_1}|^2}{4} + \frac{|n_P n_{C_2}|^2}{4} + \frac{|n_A n_{C_2}|^2}{4} + |n_P n_A n_{C_1} n_{C_2}|^2 \right)^{-1/2}. \quad (7)$$

Note that  $\{n\} = \{n_P, n_A, n_{C_1}, n_{C_2}\}$  represents all the decoherence parameters. Using this channel Alice wants to teleclone an arbitrary state,  $|\phi\rangle_X = \alpha|0\rangle_X + \beta|1\rangle_X$ , to

Bob and Charlie. The full initial state, with the qubit to

teleclone, is simply given by

$$|\phi\rangle_{XPAC} = |\phi\rangle_X \otimes |\psi; \{n\}\rangle_{PAC}, \quad (8)$$

and the protocol works as follows.

Alice performs a modified Bell measurement [18, 20], i.e. she projects her original (X) and port (P) qubits onto the following modified Bell basis:

$$|\Phi_m^+\rangle = M(|00\rangle + m|11\rangle), \quad (9)$$

$$|\Phi_m^-\rangle = M(m^*|00\rangle - |11\rangle), \quad (10)$$

$$|\Psi_m^+\rangle = M(|01\rangle + m|10\rangle), \quad (11)$$

$$|\Psi_m^-\rangle = M(m^*|01\rangle - |10\rangle), \quad (12)$$

where  $M = 1/\sqrt{1+|m|^2}$ . We introduce, as will become clear soon, a free parameter ( $m$ ) in the protocol. It is a proper manipulation of this parameter that allows Alice to overcome the fidelity decrease due to her port qubit decoherence. Each projective measurement implemented by Alice on qubits  $X$  and  $P$  projects the ancillary and copy qubits to the state  $|R_j\rangle_{AC_1C_2}$ , with probability  $P_j$ . Here  $j = \{\Phi_m^+, \Phi_m^-, \Psi_m^+, \Psi_m^-\}$  stands for any possible measurement result obtained by Alice. Alice then sends Bob and Charlie her measurement result (two bits). Then, both parties apply the appropriate unitary transformation on their qubits,  $\{\Phi_m^+, \Phi_m^-, \Psi_m^+, \Psi_m^-\} \rightarrow \{I, \sigma_z, \sigma_x, \sigma_z \sigma_x\}$ . At the end of the protocol Bob (Charlie) ends up with the state  $\rho_{1(2),j} = \text{Tr}_{A,C_2(1)}(|R_j\rangle_{AC_1C_2}\langle R_j|)$ , which is obtained tracing out all but qubit  $C_1(2)$ . Therefore, Bob's (Char-

lie's) fidelity for this run of the protocol is  $F_{1(2),j} = \langle \phi | \rho_{1(2),j} | \phi \rangle_X$ .

### III. CHANNEL EFFICIENCY

We now turn to estimate the efficiency of the protocol employing the techniques developed in Ref. [18]. From now on  $\{n\}$  and  $m$  are all real numbers since it can be shown that we do not lose in generality by such assumptions [18]. In general the probabilities  $P_j$  and the fidelities  $F_{1(2),j}$  depend on  $\alpha$  and  $\beta$ . Moreover, Alice can change the values of  $\alpha$  and  $\beta$  of the transferred state at will for each run of the protocol. Therefore, in order to get  $\alpha$ - and  $\beta$ -independent results we average over many implementations of the protocol obtaining the *protocol efficiency* [18]

$$C_{1(2)}^{pro} = \sum_j \langle P_j F_{1(2),j} \rangle.$$

In the averaging process we will need the quantities  $\langle |\alpha|^2 \rangle$ ,  $\langle |\alpha|^4 \rangle$ ,  $\langle |\beta|^2 \rangle$ ,  $\langle |\beta|^4 \rangle$  and  $\langle |\alpha\beta|^2 \rangle$ . In Ref. [18] they were shown to be  $\langle |\alpha|^2 \rangle = \langle |\beta|^2 \rangle = 1/2$ ,  $\langle |\alpha|^4 \rangle = \langle |\beta|^4 \rangle = 1/3$ , and  $\langle |\alpha\beta|^2 \rangle = 1/6$ . We can interpret  $C^{pro}$  as the average qubit transmission rate for a given protocol choice [18].

The averaged probabilities, Bob's average fidelities, and his channel efficiency are:

$$\langle P_{\Phi_m^+} \rangle = \langle P_{\Psi_m^-} \rangle = \frac{A^2 M^2}{2} \left( 1 + \frac{n_P^2 n_{C_1}^2 m^2}{4} + \frac{n_A^2 n_{C_1}^2}{4} + \frac{n_P^2 n_{C_2}^2 m^2}{4} + \frac{n_A^2 n_{C_2}^2}{4} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 m^2 \right), \quad (13)$$

$$\langle P_{\Phi_m^-} \rangle = \langle P_{\Psi_m^+} \rangle = \frac{A^2 M^2}{2} \left( m^2 + \frac{n_P^2 n_{C_1}^2}{4} + \frac{n_A^2 n_{C_1}^2 m^2}{4} + \frac{n_P^2 n_{C_2}^2}{4} + \frac{n_A^2 n_{C_2}^2 m^2}{4} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 \right), \quad (14)$$

$$\begin{aligned} \langle F_{1,\Phi_m^+, \Psi_m^-} P_{\Phi_m^+, \Psi_m^-} \rangle &= \frac{A^2 M^2}{3} \left( 1 + \frac{n_A^2 n_{C_1}^2}{8} + \frac{n_A^2 n_{C_2}^2}{4} + \frac{n_P n_{C_1} m}{2} + \frac{n_P n_A^2 n_{C_1} n_{C_2}^2 m}{2} + \frac{n_P^2 n_{C_1}^2 m^2}{4} \right. \\ &\quad \left. + \frac{n_P^2 n_{C_2}^2 m^2}{8} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 m^2 \right), \end{aligned} \quad (15)$$

$$\begin{aligned} \langle F_{1,\Phi_m^-, \Psi_m^+} P_{\Phi_m^-, \Psi_m^+} \rangle &= \frac{A^2 M^2}{3} \left( m^2 + \frac{n_A^2 n_{C_1}^2 m^2}{8} + \frac{n_A^2 n_{C_2}^2 m^2}{4} + \frac{n_P n_{C_1} m}{2} + \frac{n_P n_A^2 n_{C_1} n_{C_2}^2 m}{2} + \frac{n_P^2 n_{C_1}^2}{4} \right. \\ &\quad \left. + \frac{n_P^2 n_{C_2}^2}{8} + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2 \right), \end{aligned} \quad (16)$$

$$C_1^{pro} = \frac{2}{3} \left( 1 + \frac{1}{2} \frac{(1+n_P^2)(1+n_{C_1}^2)(1+n_A^2 n_{C_2}^2) c(n_P) c(n_{C_1}) c(m) - (n_A^2 n_{C_1}^2 + n_P^2 n_{C_2}^2)}{(n_P^2 + n_A^2)(n_{C_1}^2 + n_{C_2}^2) + 4(1 + n_P^2 n_A^2 n_{C_1}^2 n_{C_2}^2)} \right). \quad (17)$$

Here  $c(n) = 2n/(1+n^2)$  is the concurrence [21] of the state  $1/\sqrt{1+n^2}(|00\rangle + n|11\rangle)$ . On the other hand, Charlie's fidelities and his channel efficiency are simply obtained by changing  $n_{C_1} \leftrightarrow n_{C_2}$ . For the standard tele-

cloning protocol  $\{n\} = m = 1$  and one obtains the well-known result of  $\langle P_j \rangle = 1/4$ ,  $\langle F_{1(2),j} P_j \rangle = 5/24$ , and  $C_{1(2)}^{pro} = 5/6$ , which is the optimal average fidelity [8].

We now begin to study each qubit's decoherence ef-

fect on the channel efficiency  $C^{pro}$ . We investigate how the port, ancillary and copies' decoherence influence the overall channel efficiency and how we can remedy the decoherence effect as modelled by Eq. (5).

### A. Port qubit treatment

The first qubit we treat is the port. Keeping all other qubits noiseless (no decoherence, i.e.  $n_A = n_{C_{1,2}} = 1.0$ ) we get:

$$C_{1(2)}^{pro} = \sum_j \langle F_{1(2),j} P_j \rangle = \frac{11}{18} \left( 1 + \frac{4c(m)c(n_P)}{11} \right). \quad (18)$$

Note that for  $n_P = m = 1$  we obtain  $C_{1(2)}^{pro} = 5/6$ , the original telecloning efficiency [8]. Moreover, noting that for this case the channel can be written as

$$|\psi; \{n\}\rangle_{PAC} = \frac{1}{\sqrt{1+n_P^2}} (|0\rangle_P |\phi_0\rangle_{AC} + n_P |1\rangle_P |\phi_1\rangle_{AC}), \quad (19)$$

it is evident to see that the same treatment as in the Generalized Teleportation Protocol (GTP) [18] and the Generalized Quantum State Sharing (GQSTS) [20] applies here. By simply changing the measurement basis (adjusting a proper  $m$ ) and choosing the proper acceptable measurements one can either retain unit probability of success with low fidelity ( $m = 1$ , accepting all results), or transform the protocol to a probabilistic one with optimal fidelity (5/6). For example, by choosing  $m = n_P$  we recover probabilistically [18, 20] the noiseless telecloning protocol [8]. For this choice of  $m$ , only  $|\Phi_m^-\rangle$  and  $|\Psi_m^+\rangle$  are acceptable results both of which furnishing the optimal fidelity for a given run of the protocol (no need for averaging) [18, 20].

### B. Ancillary qubit treatment

Turning our attention to the ancillary qubit's decoherence, where we consider all other qubit's to have no decoherence (i.e.  $n_P = n_{C_{1,2}} = 1.0$ ) we get

$$C_{1(2)}^{pro} = \sum_j \langle F_{1(2),j} P_j \rangle = \frac{11}{18} \left( 1 + \frac{4c(m)}{11} \right). \quad (20)$$

It is interesting to note that the ancillary decoherence has no effect on the overall channel efficiency. However, for any probabilistic protocol, in which we restrict the acceptable results for less than four Bell states, the average fidelity does depend on the ancillary's decoherence ( $n_A$ ). Finally, again we find that the overall channel efficiency is optimal for  $m = 1$ , namely  $C_{1(2)}^{pro} = 5/6$ .

### C. Copy qubit treatment

The last case to consider is the one in which the copies suffer decoherence. In this case, we assume that the port and the ancillary qubits have no decoherence (i.e.  $n_P = n_A = 1.0$ ). The channel efficiency can be rewritten as

$$C_1^{pro} = \frac{1}{2} \left( 1 + \frac{2}{3} (\kappa^{(1)} + \kappa^{(2)} c(n_{C_1}) c(m)) \right), \quad (21)$$

$$\kappa^{(1)} = \frac{1}{1+\lambda}, \quad \kappa^{(2)} = \frac{1}{1+1/\lambda}, \quad (22)$$

$$\lambda = \frac{(1+n_{C_1}^2)(1+n_{C_2}^2)}{1+n_{C_1}^2 n_{C_2}^2}. \quad (23)$$

For the second copy,  $C_2^{pro}$  is given by changing  $n_{C_1} \leftrightarrow n_{C_2}$ . As we discuss below, Eq. (21) allows us to derive a couple of interesting properties for this particular protocol. Firstly, let us analyze some trivial limiting cases. For  $m = 1$ , note that when  $n_{C_1} = n_{C_2} = 1$  we obtain, as it should be,  $C_{1(2)}^{pro} = 5/6$ , the noiseless optimal limit. Moreover, when  $n_{C_1} = n_{C_2} = 0$  we get  $C_{1(2)}^{pro} = 2/3$ . This value can be understood noting that for this case the channel is  $|\psi; \{n\}\rangle_{PAC} = |0000\rangle_{PAC}$ , i.e. we have no entanglement whatsoever. Thus the telecloning protocol can be seen as the usual teleportation protocol whose efficiency is at most 2/3 when we have pure but not entangled channels [18]. Furthermore, for the case of  $n_{C_1} = 1$ , we see that the channel efficiency of the first copy does not depend on  $n_{C_2}$ , as can be seen looking at Eq. (23). A similar argument applies for the second copy channel efficiency. This is remarkable and it means that the influence of the second (first) copy decoherence on the protocol efficiency of the first (second) copy is dependent on the first (second) copy decoherence. Finally, when  $n_{C_2} = 1$  one recovers  $C_1^{pro} = \frac{11}{18} (1 + \frac{4}{11} c(m) c(n_{C_1}))$ , similar to Eq. (18), with  $n_P \leftrightarrow n_{C_1}$ . This shows that the influence of the port qubit decoherence on  $C_1^{pro}$  is exactly the same when just the first copy decoheres. However, in contrast to the case where only the port qubit decohere, we were not able to devise a procedure by which we can increase the fidelity of the copies, even in a probabilistic protocol. In other words, equating  $m = n_{C_1}$  does not improve the fidelity of the copies, contrary to a similar successful strategy employed for the port's decoherence case ( $m = n_P$ ).

## IV. GTC TO GTP CONVERSION

We end this contribution showing how one can convert the GTC to the GTP protocol. In other words, we want to show how it is possible, using first local and then global unitary operations, to convert the GTC channel  $|\psi; \{n\}\rangle_{PAC}$  (Eq. (6)) to the GTP channel  $|\Psi_{n_{C_1}}^{GTP}\rangle = (1/\sqrt{1+n_{C_1}^2}) (|00\rangle + n_{C_1} |11\rangle)$ . We want, therefore, to create a GTP channel between Alice and copy 1 (Bob) in detriment of copy 2 (Charlie), who will have a considerable decrease of his channel efficiency. This can be achieved by 'disentangling' copy 2 from Alice's qubit and

copy 1. The final goal is to concentrate all the entanglement of the channel between Alice and Bob.

### A. Local unitary operations

Firstly, let us restrict ourselves to local unitary operations (Alice's site). If we remember that the ancillary qubit (A) is assumed to be with Alice, she can only operate on the port (P) and ancillary qubits (See Fig. 1). An optimal strategy for Alice, when we set  $n_P = n_A = 1$ ,  $n_{C_2} = 0$ , and  $m = 1$  for the measurement basis, is the application of the following unitary operation on A and P:

$$\mathbf{R}_{jk}(q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A_q & -qA_q & 0 \\ 0 & q^*A_q & A_q & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (24)$$

$$A_q = \frac{1}{\sqrt{1+|q|^2}}, \quad (25)$$

where  $j, k = P, A$  are the two qubits Alice acts upon. Here  $\mathbf{R}_{jk}$  is written in the basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$  and it is basically a rotation in the  $|01\rangle, |10\rangle$  plane. The best result (maximal channel efficiency) is achieved for the case  $q = 1$ . This choice for  $q$  gives the channel (note the order in which the qubits are written),

$$|\Psi_{n_{C_1}/\sqrt{2}}^{GTP}\rangle_{PC_1AC_2} = \frac{1}{\sqrt{1+n_{C_1}^2/2}} \times \left( |00\rangle + \frac{n_{C_1}}{\sqrt{2}} |11\rangle \right) \otimes |00\rangle. \quad (26)$$

This is a GTP-like channel between  $P$  and  $C_1$  but with  $n_{C_1}/\sqrt{2}$  instead of  $n_{C_1}$ , which is the cost one pays for the inaccessibility to the copy qubits. However, the channel efficiency is still large since for  $n_{C_1} = 1$  we have  $C_1^{pro} = (6 + 2\sqrt{2})/9 \approx 0.981$ .

Borrowing from the case of  $n_{C_2} = 0$  and any  $n_{C_1}$ , to the case of  $n_{C_1} = 1$  and  $n_{C_2} < 1$ , we can make the same transformations as before and arrive at the following channel efficiencies:

$$C_1^{pro} = \frac{6 + 2\sqrt{2} + 5n_{C_2}^2}{9(1+n_{C_2}^2)}, \quad (27)$$

$$C_2^{pro} = \frac{5 + 2\sqrt{2}n_{C_2} + 6n_{C_2}^2}{9(1+n_{C_2}^2)}. \quad (28)$$

Looking at Eqs. (27) and (28) we can draw several interesting conclusions: (i)  $C_1^{pro} > C_2^{pro}$  for all  $n_{C_2}$ , which is a consequence of the fact that Alice's qubit is more entangled with copy 1 qubit, located at Bob's, in comparison with copy 2 at Charlie's; (ii) For  $n_{C_2} = 1$  we get  $C_1^{pro} < 5/6$  and  $C_2^{pro} < 5/6$ , showing that the unitary transformation reduces the channel efficiency of the GTC

protocol when compared with the efficiency of a maximally entangled GTP channel; (iii) Eqs. (27) and (28), however, show that for  $n_{C_2} \leq \sqrt{\frac{4\sqrt{2}-3}{5}}$  one can achieve  $C_1^{pro} \geq 5/6$ , thus highlighting the transition point from the GTC to the GTP scenario.

### B. Global unitary operations

If we now allow Alice to implement global unitary operations, i.e., she has access, in addition to the port and ancillary qubits, to at least one of the copies, she is able to recover the GTP channel from the GTC channel via two transformations. We also assume, from now on, that Alice has access only to copy 1, being, thus, impossible for her to work with copy 2.

As we did before, we first assume that  $n_P = n_A = 1$ ,  $n_{C_2} = 0$ , and  $m = 1$  for the measurement basis. With this choice, the GTC channel reads,

$$|\psi; \{n\}\rangle_{PAC_1C_2} = \frac{1}{\sqrt{4+2n_{C_1}^2}} \left( 2|000\rangle + n_{C_1}|101\rangle + n_{C_1}|011\rangle \right) \otimes |0\rangle. \quad (29)$$

First Alice implements the following unitary operation on the ancillary and copy 1 qubits, setting  $q = n_{C_1}/2$ ,

$$\mathbf{T}_{jk}(q) = \begin{pmatrix} A_q & 0 & 0 & qA_q \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -q^*A_q & 0 & 0 & A_q \end{pmatrix}, \quad (30)$$

$$A_q = \frac{1}{\sqrt{1+|q|^2}}, \quad (31)$$

where  $j, k$  are the two qubits Alice acts upon and now  $\mathbf{T}$  is basically a rotation in the  $|00\rangle, |11\rangle$  plane. The resulting state,  $|\Psi^{(1)}\rangle = \mathbf{T}_{A,C_1} \left( \frac{n_{C_1}}{2} \right) |\psi; \{n\}\rangle_{PAC_1C_2}$ , is,

$$|\Psi^{(1)}\rangle_{PAC_1C_2} = \frac{1}{\sqrt{4+2n_{C_1}^2}} \left( \sqrt{4+n_{C_1}^2} |0000\rangle + n_{C_1} |1010\rangle \right). \quad (32)$$

The second transformation Alice implements are on the port and copy 1 qubits with  $q = n_{C_1}(1 - \sqrt{4+n_{C_1}^2})/(n_{C_1}^2 + \sqrt{4+n_{C_1}^2})$ . The final state,

$$|\Psi^{GTP}\rangle_{PC_1AC_2} = \mathbf{T}_{P,C_1} \left( \frac{n_{C_1}(1 - \sqrt{4+n_{C_1}^2})}{n_{C_1}^2 + \sqrt{4+n_{C_1}^2}} \right) |\Psi^{(1)}\rangle,$$

is given as,

$$|\Psi^{GTP}\rangle_{PC_1AC_2} = \frac{1}{\sqrt{1+n_{C_1}^2}} (|00\rangle + n_{C_1}|11\rangle) \otimes |00\rangle. \quad (33)$$

This is exactly the GTP channel [18, 19] we were looking for. Therefore, if Alice has also access to copy 1, it is possible to go from GTC to GTP.

Again, borrowing from the case in which  $n_{C_2} = 0$  and  $n_{C_1}$  is the free parameter, to the case of  $n_{C_1} = 1$  and  $n_{C_2} < 1$ , we can make the same transformations as before and arrive at the following channel efficiencies:

$$C_1^{pro} = \frac{135 + 77n_{C_2}^2}{135(1 + n_{C_2}^2)}, \quad (34)$$

$$C_2^{pro} = \frac{135 + (8\sqrt{5} + 159)n_{C_2}^2 + 24\sqrt{15}n_{C_2}}{270(1 + n_{C_2}^2)}. \quad (35)$$

Here, again, we have the following interesting results: (i)  $C_1^{pro} > C_2^{pro}$  for all  $n_{C_2}$ , reflecting the concentration of entanglement between port and copy 1; (ii) For  $n_{C_2} = 1$  we get  $C_1^{pro} < 5/6$  and  $C_2^{pro} < 5/6$ , showing that the transformations also reduce the channel efficiency of the GTC protocol when compared with the efficiency for the maximally entangled GTC channel; (iii) Finally, manipulating  $C_1^{pro}$ , one sees that for  $n_{C_2} \leq \sqrt{45/71}$  one can achieve  $C_1^{pro} \geq 5/6$ , thus showing the transition point from the GTC to the GTP scenario.

## V. CONCLUSION

To conclude, we have shown that the introduction of local decoherence into the quantum telecloning proto-

col results in non-trivial protocol efficiencies which depend on the specific decoherence process of the channel's qubits. We have analyzed all the three possible decoherence scenarios. Firstly, the port's decoherence can be dealt with in a probabilistic manner, similar to the approach employed for the generalized teleportation and quantum state sharing protocols. Here we can achieve the optimal fidelity for the telecloned qubits by properly rotating Alice's measurement basis. Secondly, the ancillary's decoherence has no effect on the overall average efficiency, as expected from an ancillary. Thirdly, the copies' decoherence cannot be counter attacked using the port's decoherence approach, i.e., there is no rotation on Alice's measurement basis allowing, even probabilistically, the optimal fidelity for both telecloned qubits. Finally, we have also shown how one can convert the generalized telecloning channel, either using local or global unitary operations, to the generalized teleportation channel. All these results highlight that non-maximally pure entangled channels can also be employed to the direct implementation of quantum protocols, although only probabilistically. And this suggests that a promising route for further analysis is the study of what can be done probabilistically using directly, i.e. without distillation protocols, non-maximally mixed entangled channels.

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